creases in strength downstream, approaches the shear layer generated at the LEX leading edge. The interaction causes a "kink," as observed by Thompson,[3] to develop in the shear layer. The kink, with the shear layer feeding into it, later develops into V_4 further downstream. As a result, an off-surface saddle point S_p exists, and the shear layer generated at the LEX leading edge no longer feeds vorticity into the primary V_1. A similar surface flow structure is observed from wind-tunnel tests carried out at $M = 0.6$ and 25 deg $\leq \alpha \leq$ 35 deg.

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Minimum Sink-Speed in Power-Off Glide

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Nomenclature

A = wing aspect ratio, $(\text{span})^2/S = b^2/S$
C_D = airplane drag coefficient, $D/(\frac{1}{2}\rho V^2 S)$
C_L = airplane lift coefficient, $L/(\frac{1}{2}\rho V^2 S)$
D = drag force parallel to flight path
e = Oswald const. for best fit to drag polar data, Eq. (1)
$\mathring{h}$ = sink-speed, dh/dt
L = lift force perpendicular to flight path
S = wing planform area
V = velocity along flight path
W = airplane weight
δ = $C_{De}/(\pi Ae)$
θ = glide angle below horizontal
ρ = atmospheric mass density

Subscripts

mp = minimum power required for steady level flight
* = flight conditions for maximum, lift/drag

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Introduction

SOME aeronautical engineers who have participated in glider competition have insisted that the minimum sink-speed $\mathring{h}$ occurs at a trim velocity that is less than V_{mp}, which is the velocity corresponding to minimum power required for steady level flight. It is usually assumed that the trim lift coefficient $C_{L,mp}$ gives the minimum sink-speed for any airplane in power-off glide. Without extensive wind-tunnel or flight data, V_{mp} can be estimated as $V_*/3^{1/4}$, where V_* is the trim velocity that produces the minimum glide angle θ_* in a steady glide with zero thrust. The following analysis uses the theoretical drag polar to derive a new relation that proves that the minimum sink-speed does occur with a trim lift coefficient that is slightly greater than $C_{L,mp}$.

Analysis

The well-known airplane drag polar may be written as

$$C_D = C_{D_e} + C_L^2/\pi Ae \tag{1}$$

where the zero lift drag coefficient C_{D_e}, and the airplanes effective aspect ratio $Ae = eb^2/S$, should be selected so as to provide the best straight line approximation for wind-tunnel or flight data when plotted on a graph of C_D vs C_L^2. As shown by Laitone,[1] C_{D_e} is usually slightly less than minimum drag coefficient found in wind-tunnel tests, and the Oswald effective aspect ratio factor is usually $1 > e > 0.9$ for properly designed aircraft, in the normal flight range, well above the stalling speed.

The maximum lift-drag ratio from Eq. (1) is given by the lift coefficient C_{L_*} obtained from

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L}\right) = -\frac{C_{D_e}}{C_{L_*}^2} + \frac{1}{\pi Ae} = 0$$
$$C_{L_*} = (\pi Ae C_{D_e})^{1/2} \quad \text{and} \quad C_{D_*} = 2C_{D_e} \tag{2}$$

In a steady-state glide with zero thrust, the glide angle θ is given by Fig. 1 as

$$\theta = \tan^{-1}(C_D/C_L) = \sin^{-1}(\mathring{h}/V_\infty) = \cos^{-1}(L/W)$$
$$V_\infty = (W\cos\theta/\tfrac{1}{2}\rho SC_L)^{1/2} \quad \text{and} \quad \mathring{h} = DV_\infty/W \tag{3}$$

The trim lift coefficient C_L is constant during the steady-state glide at flight path velocity V_∞.

Obviously the minimum glide angle is given by the maximum lift-drag ratio at $C_L = C_{L_*}$ as

$$\theta_* = \tan^{-1}(C_{D_*}/C_{L_*}) = \tan^{-1}2(C_{D_e}/\pi Ae)^{1/2} \tag{4}$$

However, the minimum sink-speed is related to the minimum power mp $= D_{mp}V_{mp}$, rather than the maximum lift-drag ratio that gave the minimum glide angle. The equivalent to the power in a steady-state glide at the constant velocity V_∞ varies with the glide angle as

$$DV_\infty = \tfrac{1}{2}\rho SC_D V_\infty^3 = W^{3/2}(\tfrac{1}{2}\rho S)^{-1/2}C_D C_L^{-3/2}(\cos\theta)^{3/2} \tag{5}$$

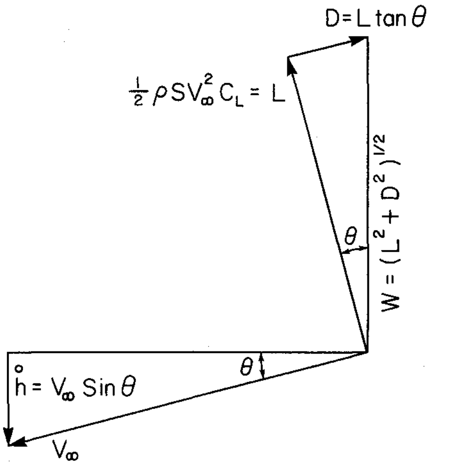

Fig. 1 Glide angle θ and sink-speed $\mathring{h}$ with zero thrust.

Then for steady level flight, when the thrust is equal to the drag so that $\theta = 0$, mp is given by differentiating Eq. (5), with C_D given by Eq. (1), to obtain

$$\frac{d(DV_\infty)}{dC_L} = 0 = \frac{d}{dC_L}(C_D C_L^{-3/2}) = -\tfrac{3}{2}C_{D_e}C_L^{-5/2} + \frac{C_L^{-1/2}}{2\pi Ae}$$

$$C_{L,\text{mp}} = (3\pi Ae C_{D_e})^{1/2} = 3^{1/2}C_{L_*}$$

$$\text{and} \quad C_{D,\text{mp}} = 4C_{D_e} = 2C_{D_*} \tag{6}$$

$$(V_{\text{mp}}/V_*) = (C_{L_*}/C_{L,\text{mp}})^{1/2} = 3^{-1/4}$$

$$\text{and} \quad (L/D)_{\text{mp}} = (\sqrt{3}/2)(L/D)_*$$

However, these relations are only valid for $\theta = 0$, for steady level flight at trim conditions given by $V_\infty = V_{\text{mp}}$ and $C_L = C_{L,\text{mp}}$. In any power-off glide, $\cos\theta = C_L(C_L^2 + C_D^2)^{-1/2}$, so that the minimum sink-speed would be given by

$$\frac{d}{dC_L}(C_D C_L^{-3/2}\cos^{3/2}\theta) = 0 = \frac{d}{dC_L}\left[\frac{C_D}{(C_L^2 + C_D^2)^{3/4}}\right]$$

$$= (C_L^2 + C_D^2)^{-3/4}\left(\frac{2C_L}{\pi Ae}\right) - (\tfrac{3}{2})(C_D C_L)\left(1 + \frac{2C_D}{\pi Ae}\right)$$

$$\times (C_L^2 + C_D^2)^{-7/4} \tag{7}$$

By introducing C_D from Eq. (1), $dC_D/dC_L = 2C_L/\pi Ae$, there finally is obtained the quadratic equation:

$$(C_L^2/\pi Ae)^2 - [\tfrac{1}{2} - (2C_{D_e}/\pi Ae)]C_L^2 + (\tfrac{3}{4}\pi Ae C_{D_e} + C_{D_e}^2) = 0 \tag{8}$$

The exact solution of this quadratic equation in C_L^2 is given by

$$C_L^2 = \left(\frac{\pi Ae}{2}\right)^2\left[1 - \frac{4C_{D_e}}{\pi Ae} - \left(1 - \frac{32C_{D_e}}{\pi Ae}\right)^{1/2}\right]$$

$$= 3\pi Ae C_{D_e} + 32C_{D_e}^2 + (512C_{D_e}^3/\pi Ae) + 0(C_{D_e}^4) \tag{9}$$

Therefore, the minimum sink-speed occurs when the trim lift coefficient is given by

$$C_{L,m} = C_{L,\text{mp}}[1 + 16\delta/3 + 640\delta^2/9 + 0(\delta^3)] \tag{10}$$

where

$$\delta = \left(\frac{C_{D_e}}{\pi Ae}\right) = \tfrac{1}{4}(D/L)_*^2 << 1$$

Then from Eqs. (1) and (6) one can obtain by neglecting δ^3

$$C_{D,m} = C_{D,\text{mp}}(1 + 8\delta + 128\delta^2)$$

$$V_m = V_{\text{mp}}(C_{L,\text{mp}}/C_{L,m})^{1/2} = V_{\text{mp}}(1 + 8\delta/3 + 32\delta^2)^{-1} \tag{11}$$

$$(L/D)_m = (L/D)_{\text{mp}}(1 - 8\delta/3 - 320\delta^2/9)$$

For all practical purposes it is unnecessary to include the δ^2 terms for the numerical calculation of $C_{L,m}$ or $C_{D,m}$, since $\delta^2 < 10^{-4}$ when $(L/D)_* > 5$. However, the δ^2 terms are essential for the evaluation of $\mathring{h}$ since the δ terms all cancel, leaving only δ^2, and the higher-order terms. The simplest procedure for calculating $\mathring{h}$ is to express Eq. (5) in terms of (C_D/C_L) by noting that $L = W\cos\theta$, so that $\cos\theta = (1 + C_D^2/C_L^2)^{-1/2}$

$$\mathring{h} = DV_\infty/W = V(D/L)(\cos\theta)^{3/2} = V(C_D/C_L)(1$$

$$+ C_D^2/C_L^2)^{-3/4} = V(C_D/C_L)[1 - (\tfrac{3}{4})(C_D/C_L)^2$$

$$+ (\tfrac{21}{32})(C_D/C_L)^4 + 0(C_D/C_L)^6] \tag{12}$$

For the minimum sink-speed $\mathring{h}_m$, the velocity $V = V_m$ is the steady glide trim-speed corresponding to $C_{L,m}$ and $C_{D,m}$, which remain constant when the steady-state velocity V_m, at glide angle θ_m, is attained, after the power is cut off. Similarly, the sink-speed $\mathring{h}_{\text{mp}}$ corresponds to the steady glide trim-speed V_{mp}, with $C_{L,\text{mp}}$ and $C_{D,\text{mp}}$ fixed by Eq. (6) in terms of C_{D_e} and πAe so that

$$\mathring{h}_m/\mathring{h}_{\text{mp}} = 1 - (736/27)\delta^2 + \mathcal{O}(\delta^3) \tag{13}$$

where, as before

$$\delta = \frac{C_{D_e}}{\pi Ae} = \tfrac{3}{16}(D/L)_{\text{mp}}^2 = \tfrac{1}{4}(D/L)_*^2 << 1 \tag{14}$$

This proves that as long as the airplane drag can be approximated by Eq. (1), the decrease in $\mathring{h}_{\text{mp}}$ is negligible, since $1 > (\mathring{h}_m/\mathring{h}_{\text{mp}}) \geq 0.9973$ for $(L/D)_* \geq 5$ and $\delta \leq 1/100$. For conventional airplanes the increases in C_L and C_D are also negligible, as is easily shown by the following calculations for a low performance light airplane having $(L/D)_* = 10$, $\pi Ae = 16$, $C_{D_e} = 0.04$, $\delta = 1/400$ and $C_{L,\text{mp}} = 1.386$:

$$C_{L,m} = C_{L,\text{mp}}(1.01333 + 4.444 \times 10^{-4}) = 1.405$$

$$C_{D,m} = C_{D,\text{mp}}(1.02 + 8 \times 10^{-4}) = 0.1633$$

$$V_m = V_{\text{mp}}(1.00667 + 2 \times 10^{-4})^{-1} = 0.9932V_{\text{mp}}$$

Even though the δ^2 terms are not necessary, they have been included to show that Eq. (11) agrees with the exact solution from Eq. (9), to four significant figures.

For the typical sailplane used in glider competitions $(L/D)_* \approx 40$; consequently, the increase in $C_{L,m}$ over $C_{L,\text{mp}}$, as defined by Eqs. (1) and (9), is trivial. Therefore, any measurable differences must be entirely caused by the sudden increase in the drag, as the velocity approaches the stall speed. Since $C_{L_*} \approx 0.9$ for $C_{D_e} \approx 0.011$ and $A = (b^2/S) \approx 25$ for the typical competition sailplaine, Eq. (6) suggests that $C_{L,\text{mp}} \approx 1.56$ is either already in stalled flight, or entering a regime of suddenly increasing drag. In any case Eqs. (1) and (9) are no longer applicable. However, since the drag is greater than that prediced by Eq. (1), the actual sink-speed has to be greater than the $\mathring{h}_{\text{mp}}$ prediced by Eq. (12), with $V = V_{\text{mp}}$, and $(C_D/C_L) = (C/L)_{\text{mp}}$ from Eq. (6).

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